

MAGNEIC EFFECT ON FLUID FLOW IN A ROTATING STRAIGHT DUCT WITH RECTANGULAR CROSS SECTION

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ABSTRACT

Numerical study is performed to investigate the Magnetohydrodynamic incompressible viscous steady flow through a duct with rectangular cross-section to examine the combined effect of Coriolis force and aspect ratio. The flow depends on the magnetic parameter M_g , Dean number D_n and rotation parameter R . Spectral method is applied as a main tool for the numerical technique; where Chebyshev polynomials, Collocation methods, and Iteration method are used as secondary tools. The flow patterns have been shown graphically for large Dean Numbers as well as magnetic parameter and a wide range of magnetic parameter. Two vortex solutions have been found. Axial velocity has been found to increase with the increase of Dean Number and decrease with the increase of magnetic parameter. For high magnetic parameter & Dean Number and small Taylors number almost all the fluid particles strength are weak.

Keywords: Dean Number, Magnetic Parameter and Rotation Parameter

1. INTRODUCTION

When a straight pipe rotates at constant angular velocity about the axis of its cylindrical symmetry, the fluid flowing in it is subjected to both a Coriolis and a Centrifugal force. Such rotating passages are used in cooling system for conductors of electric generators and generator motors for pumped –storage stations. Flow in rotating straight pipe is of interest because the secondary flows in this case are qualitatively similar to those in stationary curved systems in view of the similar centrifugal mechanism including the secondary flows in two systems Ishigaki [1]. The earliest work on the flow in a rotating straight pipe was carried out for the asymptotic limits of weak and strong rotations by Barua and Benton and Baltimore [2] using a perturbation expansion to the Hgen-Poiseulle flow. The study of Mori and Nakayama[3], Ito and Nanbu [4] and Wanger and Velkoff [5] for small rotational speed and high axial pressure gradient resulted good agreement with experiments, showing an increase in friction factor with rotational speed. The numerical study on rotating straight pipe with circular cross-section by Duck [6], Mansur [7] and Lei and Hsu [8] did not reveal multiplicity features, although they recognized a strong similarity between flows in curved pipes and on rotating pipes. The numerical study of the flow in a rotating straight pipe by Sharma and Nandakumar [9] revealed multiplicity feature. The laminar flow in a straight pipe rotating at a constant angular velocity about an axis perpendicular to its own was analyzed by kleshgi and Seriven [10], Nandakumar et. al. [11] and Speizel [12] in case of square cross-sections. In a numerical study of the flow in a

rotating straight pipe, Speziale [13] demonstrated that a transition from a two cell to a four cell to a four cell structures occurs as the Rossby number is changed. Flow through a rotating straight pipe with large aspect ratio by Alam, Begum, and Yamamoto [14]. Since there is no study regarding rotating rectangular straight duct in the presence of magnetic field.

Hence our aim is to study that the effects of rotation on the fully developed steady laminar flow through a rotating rectangular straight duct with magnetic field along the center line direction. Numerical solutions are obtained by using the Spectral method as a main tool and Chebyshev polynomial is used as a secondary tool.

2. GOVERNING EQUATION

The basic equation for steady-state laminar flow is continuity equation becomes:

$$\nabla \cdot \mathbf{q} = 0 \quad (1)$$

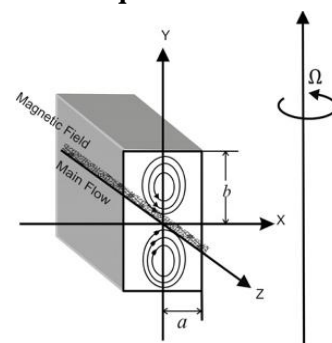


Fig 1. Geometrical configuration of rotating rectangular straight duct with magnetic field.

The left side is the inner wall and the right side is the outer wall and 'a' is the half width of the cross-section and 'b' is the half length of the duct in Figure 1.

The governing equation for the laminar flow of an incompressible viscous fluid in a rotating straight duct as follows:

$$\frac{\partial \mathbf{q}'}{\partial t} + \mathbf{q}'(\mathbf{q}' \cdot \bar{\nabla}) = -\frac{1}{\rho}(\bar{\nabla} \cdot \mathbf{p}') + \nu \bar{\nabla}^2 \mathbf{q}' + 2(\boldsymbol{\Omega} \wedge \mathbf{q}') + \frac{1}{\rho}(\mathbf{J} \wedge \mathbf{B}) \quad (2)$$

where J is the electric current density, B is the magnetic induction, ν is the kinematic viscosity and the axis of rotation is perpendicular to the span of the pipe .

The Navier –Stokes equation and Continuity equation which takes the form as:

$$u' \frac{\partial u'}{\partial x'} + v' \frac{\partial u'}{\partial y'} = -\frac{1}{\rho} \frac{\partial p'}{\partial x'} + \nu \left(\frac{\partial^2 u'}{\partial x'^2} + \frac{\partial^2 u'}{\partial y'^2} \right) - 2\Omega w' - \frac{\sigma' B_0^2}{\rho} u' \quad (3)$$

$$u' \frac{\partial v'}{\partial x'} + v' \frac{\partial v'}{\partial y'} = -\frac{1}{\rho} \frac{\partial p'}{\partial y'} + \nu \left(\frac{\partial^2 v'}{\partial x'^2} + \frac{\partial^2 v'}{\partial y'^2} \right) - \frac{\sigma' B_0^2}{\rho} v' \quad (4)$$

$$u' \frac{\partial w'}{\partial x'} + v' \frac{\partial w'}{\partial y'} = -\frac{1}{\rho} \frac{\partial p'}{\partial z'} + \nu \left(\frac{\partial^2 w'}{\partial x'^2} + \frac{\partial^2 w'}{\partial y'^2} \right) + 2\Omega u' \quad (5)$$

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0 \quad (6)$$

Now, the dependent and independent variables are then normalized as follows:

$$u' = \frac{\nu}{a} u, \quad x' = xa, \quad p' = \frac{\nu^2}{a^2} \rho p, \quad v' = \frac{\nu}{a} v, \quad y' = ya, \quad w' = \frac{\nu}{a} w, \quad z' = 0.$$

where the variables with prime are dimensional quantities and a be the half width of the cross section of the pipe.

The non dimensional equations are as follows:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - Ru - M \quad (7)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - Mv \quad (8)$$

$$u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} = D_n + \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) + Ru \quad (9)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (10)$$

where, Rotating parameter $R = 2 \left(\frac{a^2 \Omega}{\nu} \right)$, Magnetic

parameter $M_g = \frac{\sigma'}{\mu_e} a^2 B_0^2 = \sigma' \mu_e a^2 H_0^2$ and pressure

$$\text{driven parameter } D_n = \frac{Ga^3}{\rho \nu^2}.$$

The boundary conditions are that the velocities are zero at $x = \pm 1$ and $y = \pm \left(\frac{b}{a} \right) = \pm \gamma$ (aspect ratio).

Now introduce the new variable $\bar{y} = \left(\frac{y}{a} \right)$, where γ is the aspect ratio i.e. $\gamma = \left(\frac{b}{a} \right)$, where b be the half height of

the cross- section and $u = -\left(\frac{\partial \psi}{\partial y} \right)$ and $v = \left(\frac{\partial \psi}{\partial x} \right)$ which

satisfies the equation (10).

The basic equations for ψ and w are as follows:

$$\begin{aligned} \frac{\partial^4 \psi}{\partial x^4} + \frac{2}{\gamma^2} \frac{\partial^4 \psi}{\partial \bar{y}^2 \partial x^2} + \frac{1}{\gamma^4} \frac{\partial^4 \psi}{\partial \bar{y}^4} = -\frac{1}{\gamma^3} \frac{\partial \psi}{\partial \bar{y}} \frac{\partial^3 \psi}{\partial \bar{y}^2 \partial x} \\ + \frac{1}{\gamma^3} \frac{\partial \psi}{\partial x} \frac{\partial^3 \psi}{\partial \bar{y}^3} - \frac{1}{\gamma} \frac{\partial \psi}{\partial \bar{y}} \frac{\partial^3 \psi}{\partial x^3} + \frac{1}{\gamma} \frac{\partial \psi}{\partial x} \frac{\partial^3 \psi}{\partial x^2 \partial \bar{y}} \\ - \frac{1}{\gamma} \frac{\partial w}{\partial \bar{y}} R + \left(\frac{1}{\gamma^2} \frac{\partial^2 \psi}{\partial \bar{y}^2} + \frac{\partial^2 \psi}{\partial x^2} \right) M \end{aligned} \quad (11)$$

$$\begin{aligned} \frac{\partial^2 w}{\partial x^2} + \frac{1}{\gamma^2} \frac{\partial^2 w}{\partial \bar{y}^2} = -\frac{1}{\gamma} \frac{\partial \psi}{\partial \bar{y}} \frac{\partial w}{\partial x} + \frac{1}{\gamma} \frac{\partial \psi}{\partial x} \frac{\partial w}{\partial \bar{y}} \\ - D_n + \frac{1}{\gamma} \frac{\partial \psi}{\partial \bar{y}} R \end{aligned} \quad (12)$$

The boundary conditions for ψ and w are given by

$$\begin{aligned} w(\pm 1, \bar{y}) = w(x, \pm 1) = \psi(\pm 1, \bar{y}) = 0 \\ \left(\frac{\partial \psi}{\partial x} \right) (\pm 1, \bar{y}) = \psi(x, \pm 1) = \left(\frac{\partial \psi}{\partial \bar{y}} \right) (x, \pm 1) = 0 \end{aligned}$$

Flux through straight pipe

The dimensional total flux Q' through the pipe is

$$Q' = \int_{-b-a}^b \int_{-a}^a w dx' dy' = \nu a Q \quad \text{where, } Q = \int_{-\gamma-1}^{\gamma} \int_{-1}^1 w dx d\bar{y} \text{ is}$$

the dimensionless total flux.

3. METHOD OF NUMERICAL TECHNIQUE

The theoretical study of flow in a Straight duct has been made either analytically or numerically.

The present work is based on numerical methods. For this purpose the spectral collocation method is used as a numerical technique to obtain the solution. It is necessary to discuss the method in detail. The basic ideas of the spectral and collocation method are given below. The expansion by polynomial functions is utilized to obtain steady or non-steady solution. The series of the Chebyshev polynomial is used in the x and \bar{y} directions where, x and \bar{y} are variables. Assuming the flow is symmetric along the axial direction. The expansion function $\phi_n(x)$ and $\psi_n(x)$ are expressed as:

$$\phi_n(x) = (1-x^2)T_n(x) \text{ and } \psi_n(x) = (1-x^2)^2T_n(x).$$

where, $T_n(x) = \cos(n \cos^{-1}(x))$.

$w(x, \bar{y})$ and $\psi(x, \bar{y})$ are expanded in terms of the function

$$w(x, \bar{y}) = \sum_{m=0}^M \sum_{n=0}^N w_{mn} \phi_m(x) \phi_n(\bar{y}) \quad (13)$$

$$\psi(x, \bar{y}) = \sum_{m=0}^M \sum_{n=0}^N \psi_{mn} \psi_m(x) \psi_n(\bar{y}) \quad (14)$$

where, M and N are the truncation numbers in the x and \bar{y} directions respectively. In order to obtain the solution for $w(x, \bar{y})$ and $\psi(x, \bar{y})$ the expansion series are substituted into the basic equation (11) and (12). The collocation method (Gottlieb and Orszag [15]) applied in x and \bar{y} directions yield a set of nonlinear differential equations for w_{mn} and ψ_{mn} . The collocation points are taken as (x_i, \bar{y}_j) :

$$x_i = \cos \left[\pi \left(1 - \frac{i}{M+2} \right) \right]; \quad (i = 1, 2, \dots, M+1) \quad (15)$$

$$\bar{y}_j = \cos \left[\pi \left(1 - \frac{j}{N+2} \right) \right]; \quad (j = 1, 2, \dots, N+1) \quad (16)$$

The non-linear differential equations are expanded symbolically as follows:

$$A_1 w + B_1 w + c_1 w = N_1(w_{mn}, \psi_{mn}) \quad (17)$$

$$A_2 \psi + B_2 \psi + c_2 \psi = N_2(w_{mn}, \psi_{mn}) \quad (18)$$

where, A_1, B_1, C_1 and A_2, B_2, C_2 are square matrices with $(M+1)(N+1)$ dimension. The non-linear algebraic equations thus obtained are solved by Newton-Raphson iteration method as follows:

$$w^{(p+1)} = C_1^{-1} N_1(w_{mn}^{(p)}, \psi_{mn}^{(p)}) \quad (19)$$

$$\psi^{(p+1)} = C_2^{-1} N_2(w_{mn}^{(p)}, \psi_{mn}^{(p)}) \quad (20)$$

where, p denotes the iteration number. To avoid difficulty near the point of inflection for the steady solution. For this reason, the arc-length method has been used. In the arc-length method, the arc-length 's' plays a central role in the formulation. The arc-length equation

$$\text{is: } \sum_{m=0}^M \sum_{n=0}^N \left[\left(\frac{dw_{mn}}{ds} \right)^2 + \left(\frac{d\psi_{mn}}{ds} \right)^2 \right] = 1 \quad (21)$$

which is solved simultaneously with equations (17) and

(18) by using the Newton-Raphson iteration method. An initial guess at a point $s + \Delta s$ is considered starting from point s as follows:

$$w_{mn}(s + \Delta s) = w_{mn}(s) + \frac{dw_{mn}(s)}{ds} \Delta s \quad (22)$$

$$\psi_{mn}(s + \Delta s) = \psi_{mn}(s) + \frac{d\psi_{mn}(s)}{ds} \Delta s \quad (23)$$

To obtain a correct solution at $s + \Delta s$, an iteration is carried out. The convergence is assumed by taking sufficiently small ϵ_p ($\epsilon_p < 10^{-10}$) defined as:

$$\epsilon_p = \sum_{m=0}^M \sum_{n=0}^N \left[(w_{mn}^{(p+1)} - w_{mn}^{(p)})^2 + (\psi_{mn}^{(p+1)} - \psi_{mn}^{(p)})^2 \right]$$

The basic equations and the boundary conditions allow us to get a symmetric solution with respect to the horizontal line passing through the axial direction. For sufficient accuracy, we have considered $M = 20$ and $N = 20$ in the present numerical calculations.

4. RESULT AND DISCUSSION

Fully developed flow in a rotating rectangular straight duct in the presence of magnetic field is considered for the present investigation. The main flow is forced by the magnetic field along the center line and the axis is perpendicular to the span of the duct which has been shown in the Figure 1. According to the definition of rotation parameter, the positive rotation means that the direction of rotation is in the same as the flow and it is called co-rotation and the negative rotation indicates that the rotation direction is opposite to the main flow direction and is called counter-rotation.

Steady laminar flow for viscous incompressible fluid has been analyzed under the action of large magnetic parameter (M_g) as well as Dean Number (D_n) at fixed aspect ratio (γ) and rotation parameter (R).

The main aim of this paper to find out the flow characteristics with varying magnetic parameter (M_g) while pressure driven parameter and rotational parameter remains constant. For the above mentioned purpose, consider the four cases as case 1: $D_n = 500$, case 2: $D_n = 1000$, case 3: $D_n = 1500$ and $D_n = 2000$. Thus the flow behavior of the mention for the cases has been expected since the duct rotation is involved in these cases. According to the definition of rotation parameter (R) means that the rotational direction is in the same as that of main flow.

After a comprehensive survey over the parametric space, solid curve has been obtained for the magnetic parameter (M_g) versus flux (Q) at Dean number $D_n = 500$ and Rotation parameter (R) = 50 where as aspect ratio $\gamma = 1.0$.

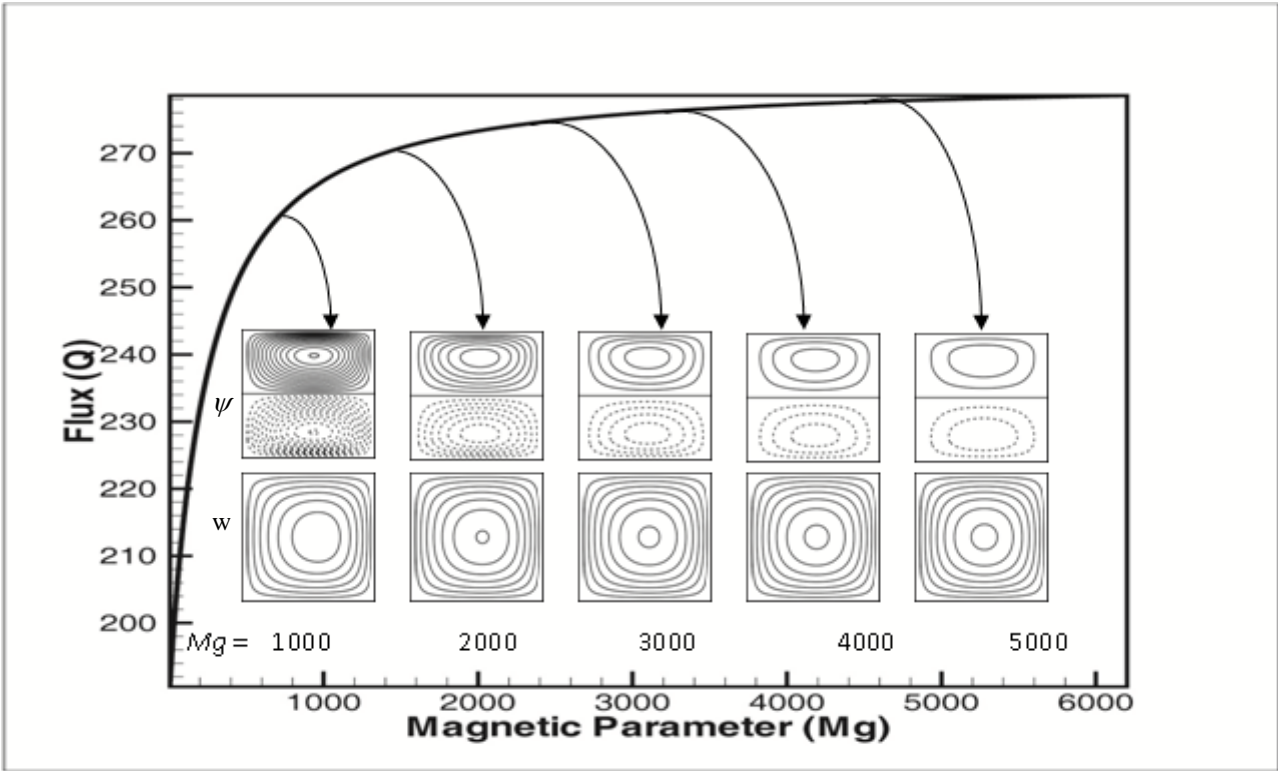


Fig 2. Stream lines of the Secondary flow (top), Contours plot of the axial flow (bottom) and solid curve for Magnetic parameter (M_g) versus Flux (Q) at Dean Number (D_n) = 500 and Rotation parameter $R = 50$.

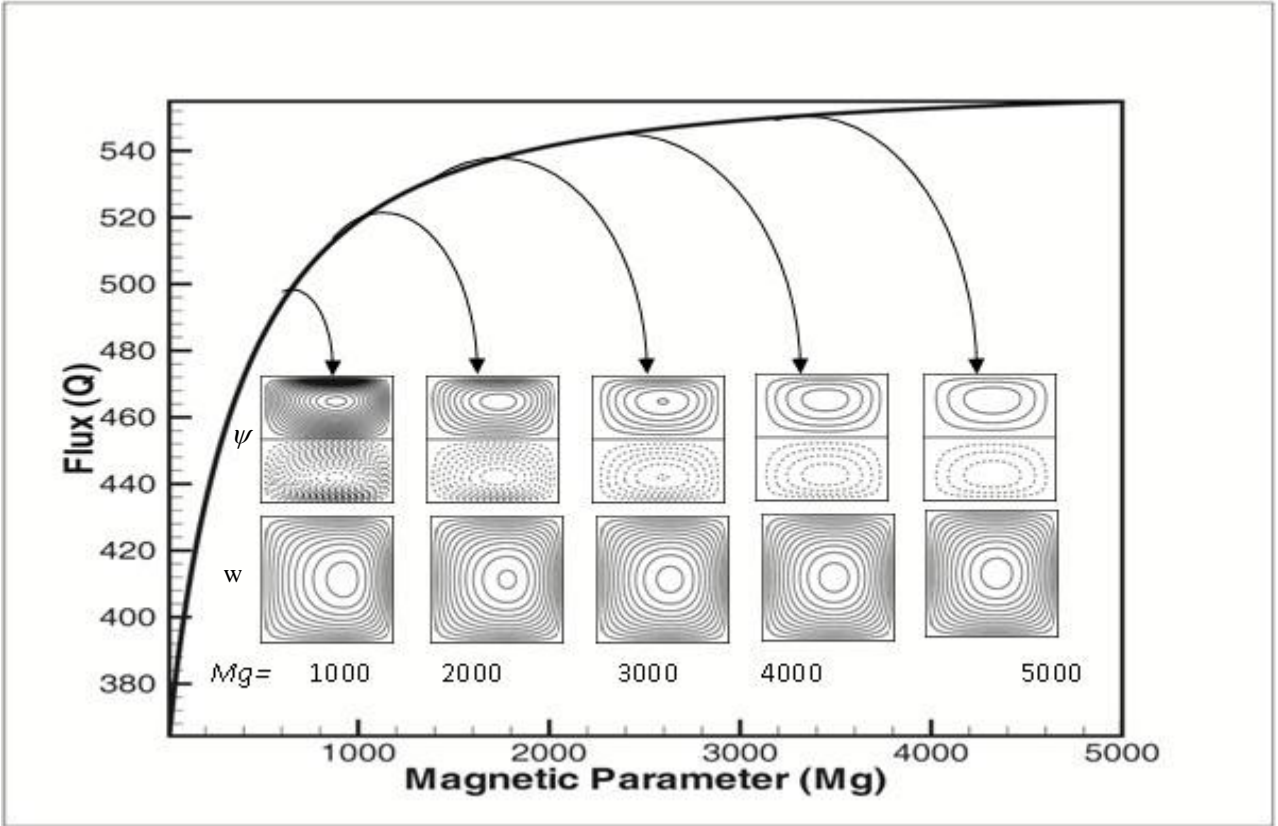


Fig 3. Stream lines of the Secondary flow (top), Contours plot of the axial flow (bottom) and solid curve for Magnetic parameter (M_g) versus Flux (Q) at Dean Number (D_n) = 1000 and Rotation parameter (R) = 50.

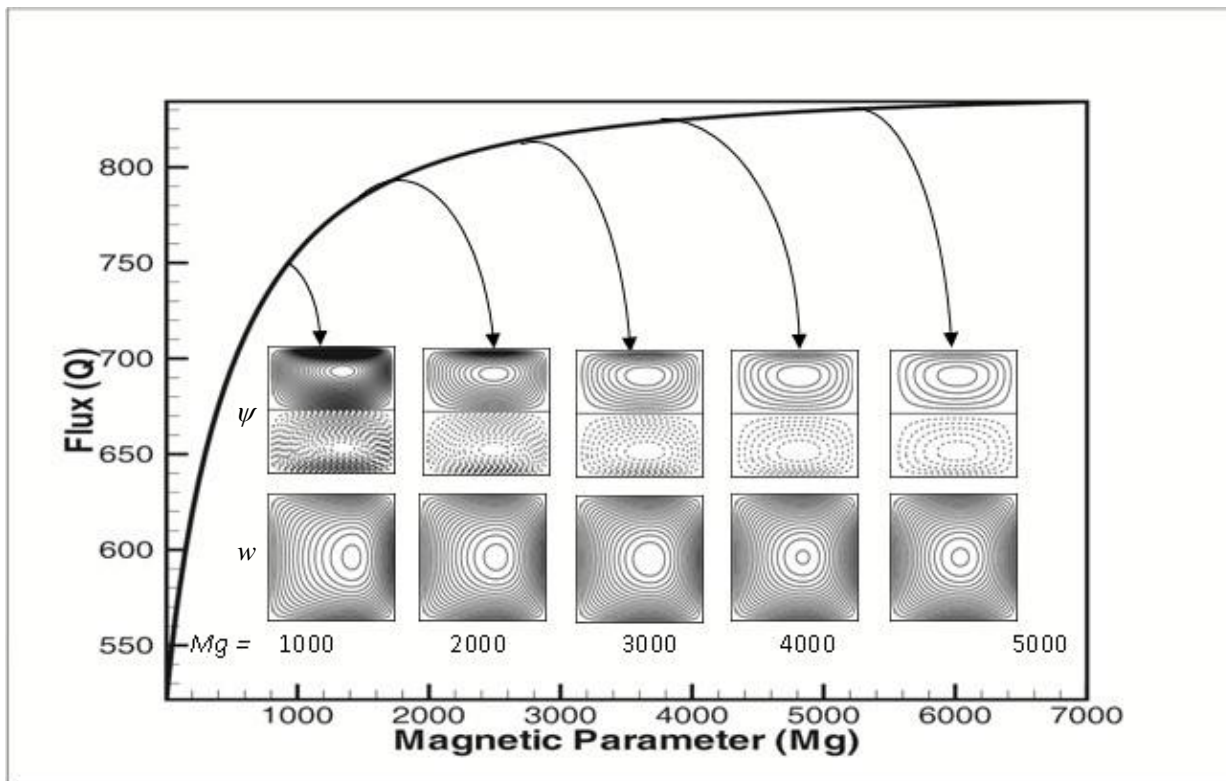


Fig 4. Stream lines of the Secondary flow (top), Contours plot of the axial flow (bottom) and solid curve for Magnetic parameter (M_g) versus Flux (Q) at Dean Number (D_n) = 1000 and Rotation parameter (R) = 50.

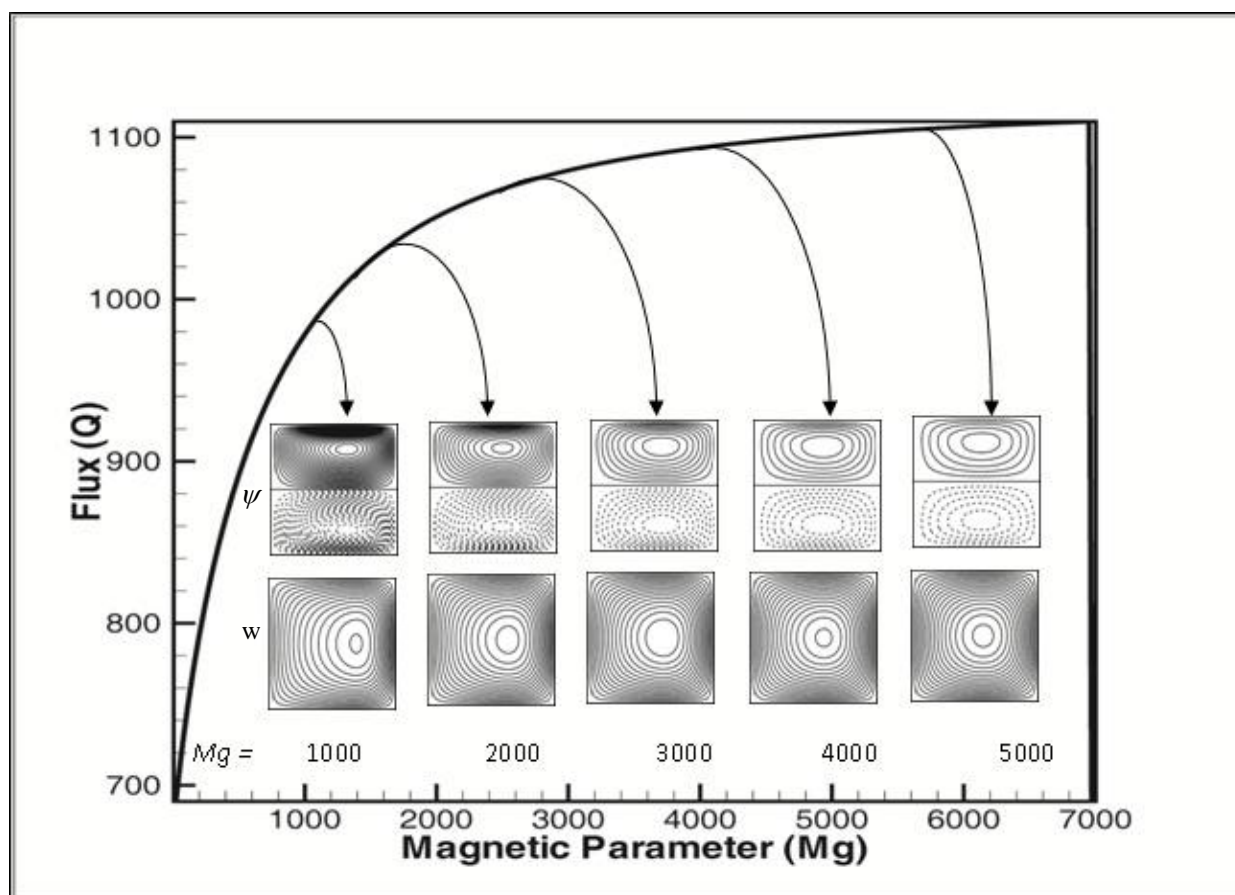


Fig 5. Stream lines of the Secondary flow (top), Contours plot of Axial flow (bottom) at Dean Number (D_n) = 2000 and Rotation parameter (R) = 50 and solid curve for Magnetic Parameter (M_g) versus Flux (Q).

Now the variation of the secondary flow and the axial flow at several values of M_g . Therefore from these Figures, the structure of the secondary flow and the axial flow in a cross-section of the straight rectangular duct. The flow patterns are shown in Figure 2 for M_g from 1000 to 5000, where the stream lines ψ and contour plots of w are drawn with $\Delta\psi = 0.075$ and $\Delta w = 20$ respectively.

In Figure 3, the solid solution curve is obtained for $D_n = 1000$ in several values of M_g without any turning point which shows the graphically representation of the flux (Q) versus the magnetic parameter (M_g). The variation of the secondary flow and the axial flow at some different values of M_g and the stream lines ψ and contour plots of w are drawn with $\Delta\psi = 0.075$ and $\Delta w = 25$ respectively.

In Figure 4 and 5, the solid curve is also obtained by the variation of flux (Q) against the magnetic parameter M_g for Dean number $D_n = 1500$ and $D_n = 2000$ respectively. The flow patterns are shown in figure 4 for $M_g = 1000, 2000, 3000, 4000, 5000$ where, the stream lines ψ and contour plots of w are drawn with $\Delta\psi = 0.075$ and $\Delta w = 25$ respectively. Also the flow patterns in figure 5 for the several values of M_g and the stream lines ψ and contour plots of w are drawn with $\Delta\psi = 0.075$ and $\Delta w = 35$ respectively.

In the figure 2, figure 3, figure 4 and figure 5 of the secondary flow, solid lines is in the counterclockwise direction while the dotted lines shows that the flow in the clock wise direction. As a result, the fluid particles strength are weak for the secondary flow vortex. The maximum axial flow is shifted to the center from the wall with increases of the magnetic parameter (M_g).

5. CONCLUSION

From the above discussion the conclusion has been drawn as follows:

1. Two vortex solutions have been found.
2. The flow patterns of the secondary flow, the fluid particles strength are weak
3. The axial flow is shifted towards the center from the wall.
4. The total flow decreases as the magnetic parameter (M_g) gradually increase.

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7. NOMENCLATURE

Symbol	Meaning
γ	Aspect ratio
M_g	Magnetic parameter
D_n	Dean number
R	Rotating parameter

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